

Accurate Evaluation of Interharmonics of a Six Pulse, Full Wave - Three Phase AC-DC Diode Rectifier on LabVIEW

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Abstract — Interharmonics are the non-integral multiples of the system's fundamental frequency. The interharmonic components can be apprehended as the intermodulation of the fundamental and harmonic components of the system with any other frequency components introduced by the load. These loads include static frequency converters, cyclo-converters, induction motors, arc furnaces and all the loads not pulsating synchronously with the fundamental frequency of the system. The harmonic and interharmonic components inflict common damage to the system and apart from these damages the interharmonics also cause light flickering, sideband torques on motor/generator and adverse effects on transformer and motor components. To filter/compensate the interharmonic components, their accurate evaluation is essential and to achieve the same the Iterative algorithm has been proposed. The main cause of spectral leakage errors is the truncation of the time-domain signal. The proposed adaptive approach calculates the immaculate window width, eliminating the spectral leakage errors in the frequency domain and thereby the interharmonics/harmonics can be calculated accurately. The algorithm does not require any inputs regarding the system frequency and interharmonic constituents of the system. The only parameter required is the signal sequence obtained by sampling the analog signal at equidistant sampling interval.

Index Terms — Discrete Fourier Transforms (DFT), Interharmonics, Interharmonics evaluation, Hanning Window, LabVIEW, AC-DC Rectifier, Interharmonics in AC-DC Rectifier, 6- pulse Rectifier, Diode Rectifier, 3 Phase Diode Rectifier.

I. INTRODUCTION

The pronounced use of power electronic devices in power systems has resulted in increase in the occurrence of interharmonics. Interharmonics are always present in the power system due to presence of power electronic equipment. The integral multiple of the fundamental supply frequency are harmonics in voltage/current wave form. Non-integral multiple of the fundamental supply frequencies are interharmonics in the respective waveforms of voltage and current. The exact spectrum of a waveform provides a clear understanding of the causes and effects of waveform distortion. So in modern power systems, accurate analysis and measurement of the system harmonics and interharmonics component are important. In this case, the

basic tool is Discrete Fourier transform (DFT). DFT approximates the continuous Fourier transform of the time-domain signal. This approximation is a function of the waveform being analysed and the signal sequence covered by the window width. If the window width of DFT is not properly chosen there will be spectral leakage [1], [2]. In order to limit leakage effects, the classical Hanning window is used instead of the rectangular window [3] which can reduce the spectral leakage to some extent. It is well known that the best solution of avoiding spectrum leakage is to select a window width because the interharmonic frequencies are unpredictable and the system frequency may also vary [4]. Here an adaptive algorithm is shown which is used to determine the window width that covers a period through an iterative procedure based on correlation calculation. Given a signal sequence it is obtained by sampling a periodic time-domain waveform at equidistant interval. On iterative comparison of samples, suitable window width is selected which eliminates the unwanted frequency leakage caused by truncation [5]. A 6-pulse AC-DC rectifier is taken for case study which shows that the proposed adaptive approach completely eliminates or significantly reduces the spectral leakage errors of DFT in the frequency domain.

A. Spectral leakage in DFT due to truncation

If the test signal is slightly off frequency, i.e. the input signal doesn't complete a whole number of cycles within the DFT time window, a distortion called spectral leakage occurs. A small frequency error has little effect on the main signal, but has a strong effect on the DFT noise floor.

B. Period of a Sampled Signal

Let a signal

$$S(t) = \sin(2\pi f t)$$

f is the frequency and it is a rational number. f_s is the sampling frequency. The number of samples is expressed as ($N_t = f_s/f$) per period T which is a rational number and normally not an integer. Assume,

$$N_t = \frac{P_1}{P_2}$$

Where P_1 and P_2 are integers and have a common divisor of one. Now we multiply P_2 by T then $T_p = P_2 T$ which covers P_2 cycles (an integer multiple of period). The number of

period). The number of samples within the period T_p is expressed as

$$N_p = P_2 \frac{f_s}{f}$$

It is a positive integer and always covers integer multiples of period T . It can be explained as the period of the discrete-time signal $S(n)$ which is obtained by sampling the periodic signal $S(t)$.

$$\text{So } S(n) = S(n + N_p)$$

Where n is an integer. The results have no spectral Leakage if N_p samples are used for DFT. The period N_p is a function of both the signal frequency f and the sampling frequency f_s .

C. Period of a Sampled Signal with Interharmonics

Consider a signal $S(t)$, in the form

$$S(t) = \sin(2\pi f t) + \sin(2\pi f_1 t).$$

Where $f_1 = \alpha f$. In previous case α is an integer, we expressed

$$N_p = P_2 \frac{f_s}{f} \quad (1)$$

If α is a non-integer, we consider $\alpha = \alpha_1 / \alpha_2$, where α_1 and α_2 are integers and have common divisor of 1. Substitute it into $S(t)$, then

$$\begin{aligned} S(t) &= \sin(2\pi f t) + \sin(2\pi f_1 t) \\ \Rightarrow S(t) &= \sin(2\pi \alpha_2 f_p t) + \sin(2\pi \alpha_1 f_p t) \end{aligned} \quad (2)$$

Where $f_p = f / \alpha_2$ from “(2),” we observed that, there are two harmonics of a fundamental frequency of $f_p = f / \alpha_2$ the period of which is found by “(1).” When a signal has interharmonics the period N_p is a function of the signal frequency, the sampling frequency and the interharmonic components. N_p is analytically determined from a sampled waveform. But we know that if a signal sequence has a period of N_p samples, then they will repeat themselves in the next adjacent period.

D. Literature Review

DFT approximates the continuous Fourier transform of the time-domain signal as proposed by R. W. Ramirez [1]. This is a function of waveform and the signal sequence covered by window width. The importance of exact spectrum of waveform in depicting the causes and effects of waveform distortion [2]. The importance of classical Hanning Window has been depicted by A. Testa, D. Gallo, and R. Langella [3].

Calculation of Interharmonics of time varying loads by

A.K. Baliar Singh, G.C. Martha, N.R. Samal, I. Shial has formed the basis for developing the case study for AC-DC rectifier [9].

II. METHOD

The method has been demonstrated in the following segments.

A. Correlation Calculation of Discrete Time Signals

We obtain a discrete-time signal, when an analog signal is measured and recorded at equally spaced sampling interval, which is a vector having a sequence of values. The correlation of two discrete-time signals can be evaluated using inner product of two vectors.

In a vector space, an inner product operation for two finite length vectors is mathematically defined as

$$\langle f, g \rangle = \|f\| \|g\| \cos \theta \quad (3)$$

Where the angle, $\theta \in [0, \Pi]$, is the angle between the two vectors in “(3).” Correlation coefficient k is defined as

$k \in [-1, 1]$ based on (3) to quantify the degree of similarity between two signal sequences. If $k=1$ is the maximum correlation when the two signals are parallel and in the same direction, and $k=-1$ is the minimum correlation when the two signals are parallel and in the opposite direction

$$k = \frac{\langle f, g \rangle}{\|f\| \|g\|} \quad (4)$$

B. Iterative algorithm for Window Width Calculation

The adaptive window width calculation algorithm is used. It searches for the most suitable window width in terms of local periodicity (or local similarity) of a signal sequence and complies with the periodicity requirement of DFT. When the correlation of two adjacent sections does not meet the defined termination criterion, the signal length of the first section is increased by one sample, and compared with the next adjacent section of the same length. The iteration continues until the termination criterion is satisfied. Consider a discrete-time signal S of infinite size:

$$S = \{s_0 s_1 s_2 \dots s_{p-1} s_p s_{p+1} \dots s_{2p-1} s_{2p} s_{2p+1} \dots\}$$

Step 1) let from S_0 to S_{p-1} choosing for p samples from S signal as an initial value for iteration, and name it X_p

$$\text{Where } X_p = \{s_0 s_1 s_2 \dots s_{p-1}\}$$

Step 2) Choose another samples from signal S , adjacent to X_p , and name it Y_p .

$$Y_p = \{s_p s_{p+1} \dots s_{2p-1}\}$$

Step 3) Calculate the correlation between the two chosen sections X_p and Y_p using “(4),”

$$K_p = \frac{\langle X_p, Y_p \rangle}{\|X_p\| \|Y_p\|}$$

K_p is the periodicity indicator of p samples of signal sequence S .

Step 4) Test the periodicity.

If $K_p = 1$ then Y_p repeats X_p . Length of p samples is the period for the two sections. The spectral lines by DFT based on a window width of samples (or an integer multiple of samples) exactly represent the continuous Fourier transform of the time-domain signal without any frequency leakage. If $K_p \neq 1$, then let $P = P + 1$, and repeat Steps 1 to 4 until the specified termination criterion is satisfied.

C. Hanning Window

The Hanning window has a shape similar to that of half a cycle of a cosine wave. The following equation defines the Hanning window.

$$w(n) = 0.5 - 0.5 \cos \frac{2\pi n}{N} \quad (5)$$

For $n = 0, 1, 2 \dots N - 1$. Where N is the length of the window and w is the window value. The Hanning window is useful for analyzing transients longer than the time duration of the window and for general-purpose applications.

III. INTER-HARMONICS IN 6-PULSE, 3-PHASE FULL WAVE, AC-DC DIODE RECTIFIER

AC-DC Rectifier is a healthy source of interharmonics. The interharmonics are produced due to the modulation of the harmonics of the source side/AC side with the harmonics observed on the DC side.

Consider a Six pulse, Three phase full wave Diode Rectifier supplied by an infinite bus at the source end and equipped with a reactor on the DC side. Hence the DC side voltage has six ripples per period of AC fundamental frequency [6]. Hence the voltage on the AC side is given by [6] considering per unit values:

$$v(t) = \cos 2\pi f_s t \pm \sum_{k'=1}^{\infty} \frac{1}{6k' \pm 1} (\cos[(6k' \pm 1)(2\pi f_s t)]) \quad (6)$$

Where f_s is the fundamental frequency of the system and k' is the order of the harmonics. The characteristic harmonics on the AC side are given by [7]:

$$f_{AC, s} = (6k' \pm 1)f_s \quad (7)$$

The process of rectification results in harmonics on the DC side which constitute every 6th harmonic observed on the AC side fundamental frequency as follows [6]:

$$f_{DC, s} = 6m.f_s \quad (8)$$

Where f_s fundamental frequency of the system and $f_{DC, s}$ represents the harmonics on the DC side.

In a rectifier, the harmonic frequency of "(7)," is modulated by the DC side harmonic frequency. The frequency modulation can be explained by the trigonometric identity:

$$\sin \alpha \cdot \sin \beta = \frac{1}{2} [\cos(\alpha - \beta) - \cos(\alpha + \beta)] \quad (9)$$

Applying the same, the source side interharmonic frequencies can be derived as

$$f_s^h = (6(k' \pm m) \pm 1).f_s \quad (10)$$

$$= (6k \pm 1)f_s \quad (11)$$

Where f_s^h indicates the interharmonics on the source side and $k = k' + m$.

Consider a voltage waveform on the AC side of the rectifier with magnitude unity and harmonic constituents considered up to the first harmonic. The equation of the waveform would be

$$v(t) = \cos 2\pi f_s t + \frac{1}{5} * (\cos((5)(2\pi f_s t))) + \frac{1}{7} * (\cos((7)(2\pi f_s t))) \quad (12)$$

Where f_s is the system fundamental frequency and is taken in this case as $f_s = 50$ Hz. The voltage waveform has been elucidated in "Fig. 1,". The Magnitude spectrum of the supply voltage is shown in "Fig. 2,"[9]. As we can observe, the frequency peaks are observed at f_s , $5f_s$ and $7f_s$ which are 50Hz, 250Hz and 350 Hz respectively.

In a 3-Phase AC-DC diode rectifier, Output DC voltage would be [8]:

$$V_{dc} = 1.65V_s \quad (13)$$

In this case $V_s = 1$. Since the DC waveform covers every 6th harmonic, the equation on the DC side can be written as:

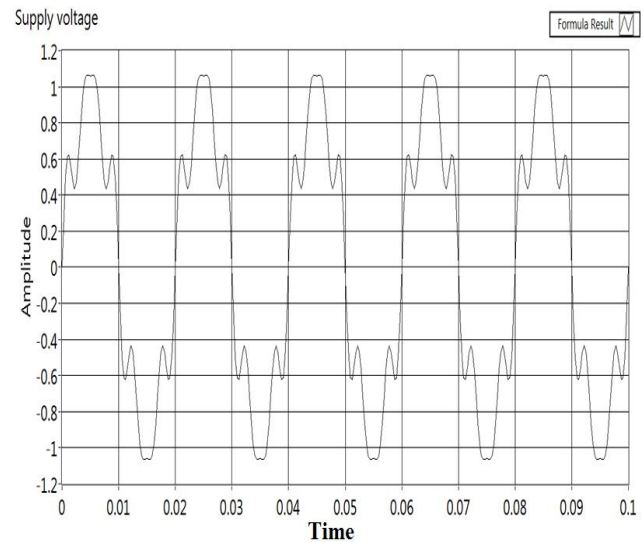


Figure 1. The Supply voltage waveform with the harmonic constituents.

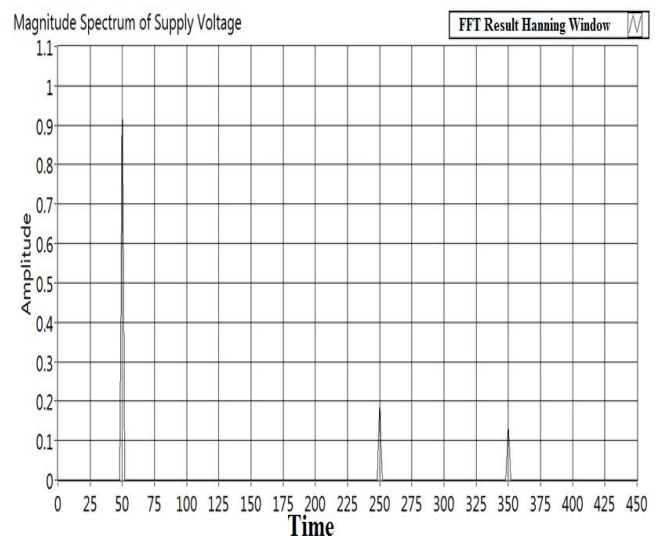


Figure 2. The magnitude spectrum of the supply voltage.

$$v_x = 1.65(1 + \cos((6)(2\pi ft)) + \cos((12)(2\pi ft)) + \cos((18)(2\pi ft)) \quad (14)$$

The intermodulation of the voltage signals at the interface of the rectifier would result in the production of the harmonic and the interharmonic components. The modulated signal is shown in “Fig. 3,”. The Magnitude spectrum of the intermodulated signal is shown in “Fig. 4,”. We observe that 50 Hz is the fundamental frequency and has the highest magnitude. The frequencies 250, 350, 550, 650, 850, 950, 1150 and 1250Hz indicate the interharmonic frequencies.

These observed interharmonics are consistent with our derived hypothesis that the interharmonics appear as in “(10),” .The window size used for spectrum analysis is 50 cycles, and sampling frequency is 3.2 kHz, which has resolution of the maximum common divider of all components i.e. 1 Hz. Sometimes, the System’s fundamental frequency might not be constant and may vary fractionally due to some changes affecting the source.

The Signal-out array gives the values of Samples of the signals at regular intervals. “Fig.5,” shows the value of first sample as 1.612219. The iterative algorithm searches for the

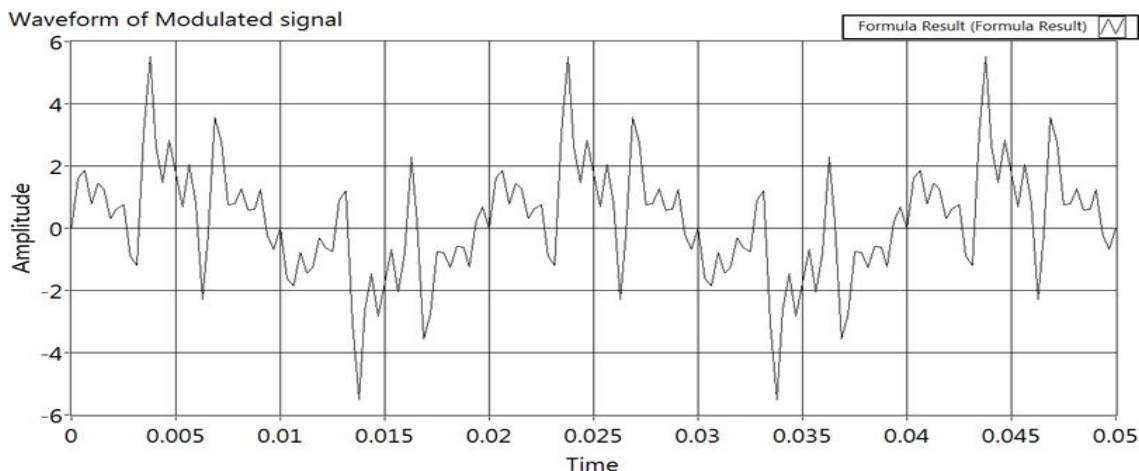


Figure 3. The Modulated waveform with the harmonic constituents.

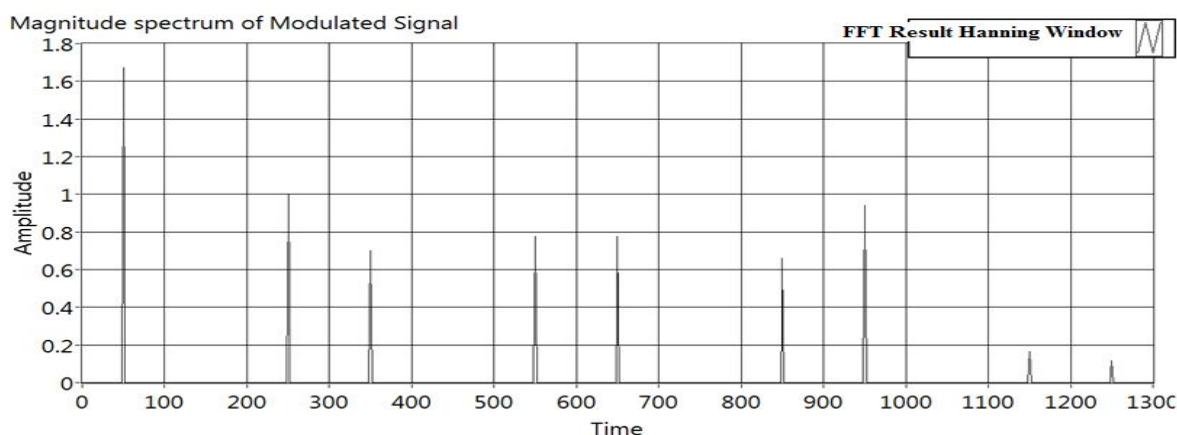


Figure 4. Magnitude Spectrum of the Modulated wave.

repetition of samples and hence helps in deciding the appropriate window for the spectral measurements.

| signal out | |
|---------------------------|----------|
| t0 | Y |
| 00:00:00 PM MM/DD/YYYY | 1 |
| dt | 1.612219 |
| 312.50u | 1.857283 |
| | 0.777376 |
| | 1.450942 |
| | 1.257363 |
| | 0.314931 |

Figure 5. The array of sample values.

| signal out | |
|---------------------------|-----------|
| t0 | Y |
| 00:00:00 PM MM/DD/YYYY | 33 |
| dt | -1.612219 |
| 312.50u | -1.857283 |
| | -0.777376 |
| | -1.450942 |
| | -1.257363 |
| | -0.314931 |

Figure 6. The array of sample values, sample value at y[33].

The algorithm evaluates that the sign opposite of this sample is found at $y[33]$ and the sample is repeated at $y[65]$, showing that the appropriate window width for the measurement of this spectrum is 64 samples and its multiples for better resolution. "Fig.6," and "Fig.7," demonstrate the results.

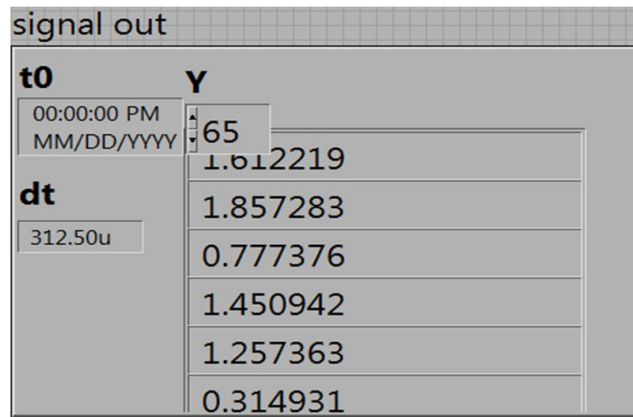


Figure 7. The array of sample values, sample value at $y[65]$.

Assuming the fundamental frequency of the system changes to 50.6 Hz, the modulated waveform of AC side voltage and DC harmonics would be as shown in "Fig. 8".

On obtaining the magnitude spectrum of the intermodulated wave with a fundamental frequency of 50.6

Hz, we note that the interharmonic components should be observed at 50.6, 253, 354.2, 556.6, 657.8, 860.2, 961.4, 1163.8 and 1265 Hz but correspondingly we observe that the interharmonics are observed at 51, 253, 354, 557, 658, 860, 961, 1164 and 1254 Hz respectively. The magnitude spectrum of the intermodulated wave at 50.6 Hz is shown in "Fig. 9".

The window size used for spectrum analysis is 50 cycles, and sampling frequency is 3.2 kHz, which has resolution of the maximum common divisor of all components i.e. 1 Hz. This size ensures that exactly integral cycles are not able to be covered in the window for all components hence leakage effect cannot be eliminated. The enlarged view of interharmonics will give a clear idea about the spectral leakage due to window width irregularities. The enlarged view[9] of the frequency at 354.2 Hz is shown in "Fig. 10,".

So the window size used for spectrum analysis is increased to 500 cycles and sampling frequency is 3.2 kHz, which has resolution of the maximum common divisor of all components, i.e. 0.1 Hz. This size ensures that exactly integral cycles are covered in the window for all components hence leakage effect can be eliminated. The magnitude spectrum of the modulated wave at frequency 50.6 Hz with 500 cycle window width is shown in "Fig. 11,".

The enlarged view[9] of the frequency at 354.2 Hz using 500 cycle window width is shown in "Fig. 12,".

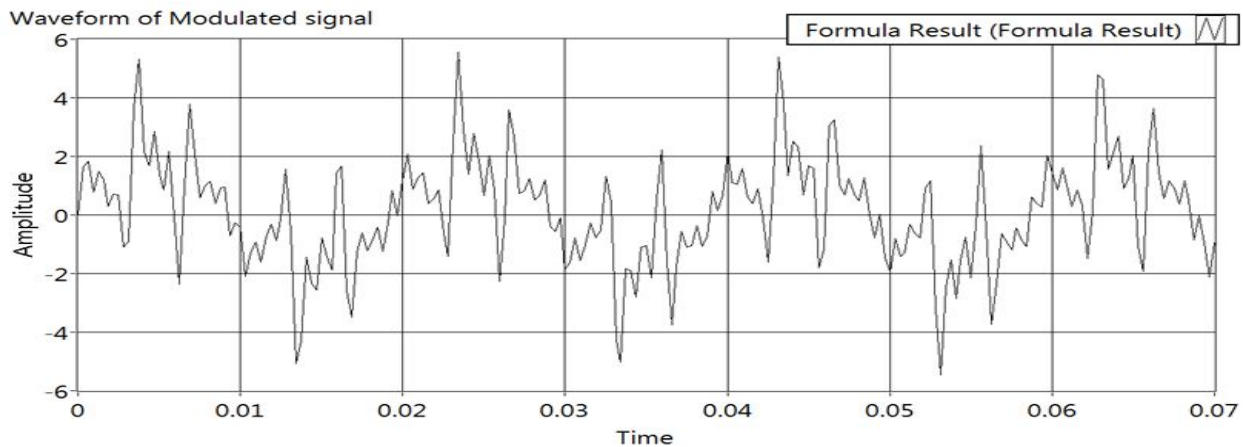


Figure 8. The intermodulated wave at frequency of 50.6 Hz.

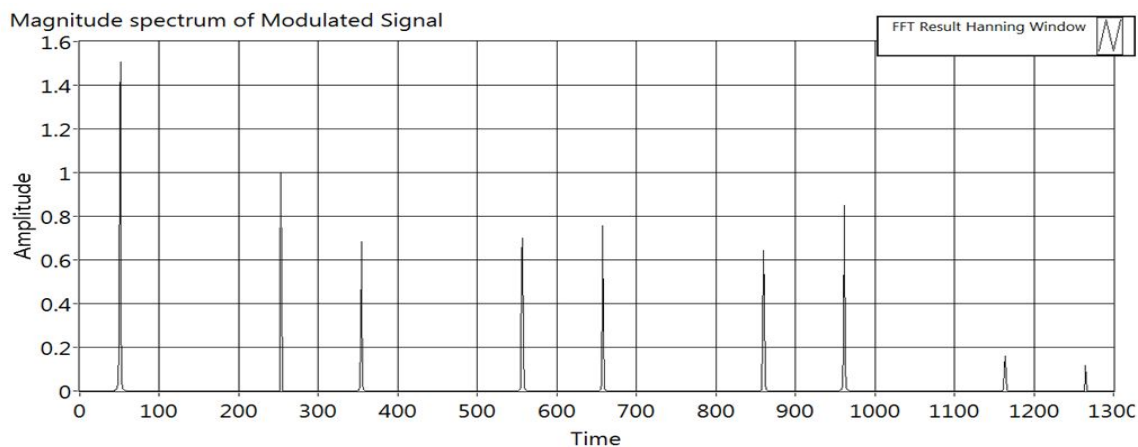


Figure 9. Magnitude Spectrum of the Modulated wave at 50.6 Hz frequency, 50 cycle window width.

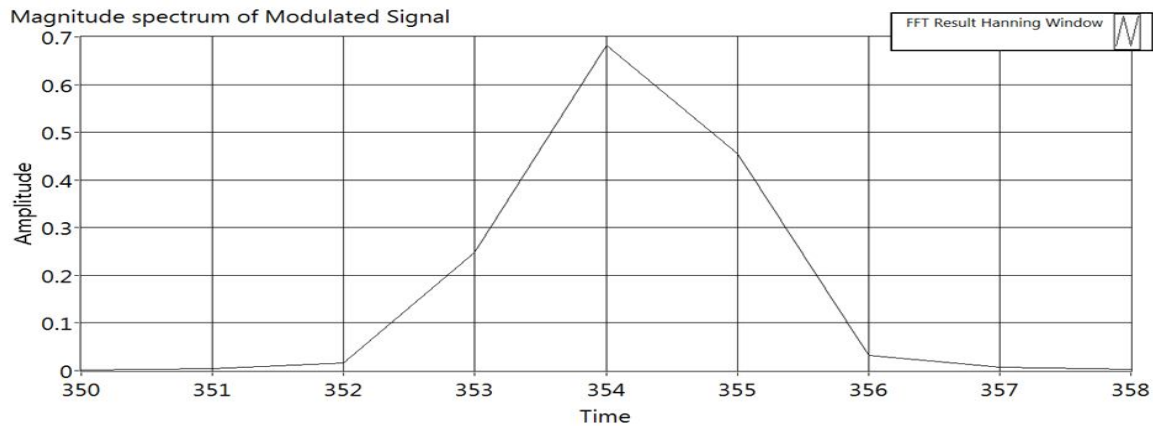


Figure 10. Enlarged view of interharmonics at 354.2 Hz with 50.6 Hz frequency, 50 cycle window width.

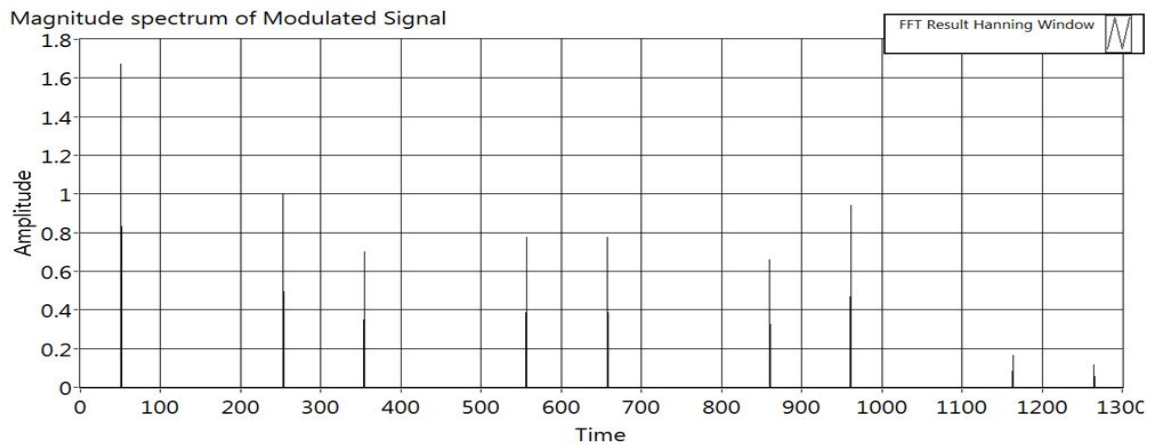


Figure 11. Magnitude Spectrum of the Modulated wave at 50.6 Hz frequency, 500 cycle window width.

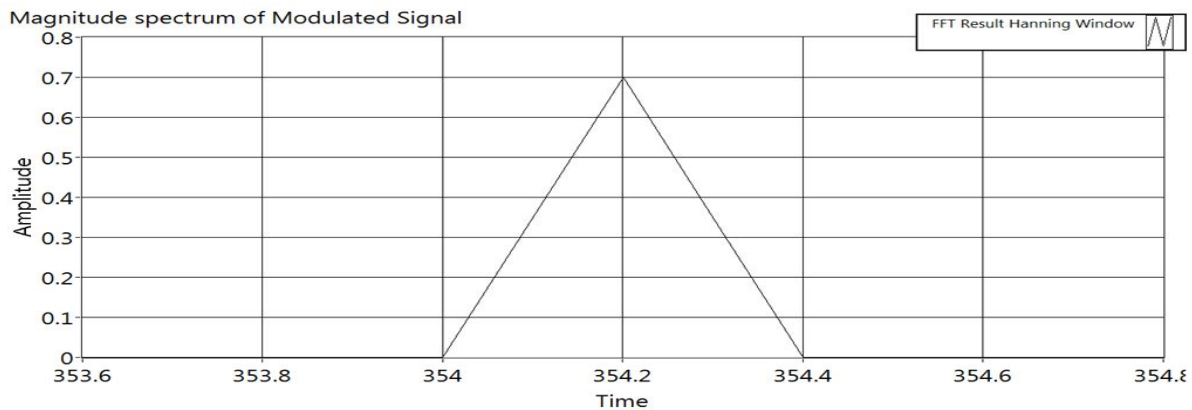


Figure 12. Enlarged view of interharmonics at 354.2 Hz with 50.6 Hz frequency, 500 cycle window width.

IV. CONCLUSIONS

Using adaptive window width the numerical method is used for exact calculation of harmonics/ interharmonics component. This method adaptively adjusts the window width based on correlation calculation thereby eliminating the unwanted spectral leakage caused by truncation. The iterative algorithm does not require any knowledge about the system frequency and the interharmonic constituents. The only parameter needed is the signal sequence obtained by sampling the analog signal at equidistant sampling interval. The case study on a Six pulse, Three phase full wave AC-DC

diode rectifier yielded comprehensive results demonstrating the importance of window width selection and thereby emphasizing the utility of the algorithm in figuring out the same.

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